Quantitative Assessment Topics

Fuente:
http://www.math.sfu.ca/ugrad/calctest/calctopics.shtml . Acceso: 2010.08.20 [ECB]

The following list of objectives was developed during the designing of the SFU's Calculus Readiness Test. The purpose of the test is to determine whether incoming students have the necessary background to succeed in the first Calculus course they will be taking.

We placed an objective on the list below if it satisfied two criteria: 1) it was stated or implied in the B.C. high school curriculum documents; and 2) the objective is relevant for Calculus courses offered at SFU. Note that the first sixty four objectives on this list also appear as objectives for the SFU Q Placement Test. While not all of these may be tested on the Calculus Readiness Test, they represent, nevertheless, knowledge required to succeed in a Calculus course.

Some of the objectives are quite general, while others are more specific. In an effort to keep the list to a manageable number, we have attempted to refrain from being very specific except in those areas where experience suggests such specificity would be helpful.

The actual test has 30 questions, and will therefore only cover a subset of the list. However, we anticipate that the assessment test will evolve and change as time goes on, and we included items as objectives whether or not they appear on the first version of the test. Some test questions may be randomly drawn from a pool of possible questions of comparable difficulty but covering different objectives on the list.

1. Use fractions, decimals, and percents to solve problems;
2. Display skills such as simplifying a complex fraction and finding a percent equivalent to two or more sequentially applied percents;
3. Compare and order fractions, decimals, and percents and find their appropriate locations on a number line;
4. Interpret percent values greater than 100%;
5. Use ratios and proportions to represent quantitative relationships;
6. Solve problems involving proportions, such as scaling and finding equivalent ratios;
7. Use factors, multiples, prime factorization, and relatively prime numbers to solve problems;
8. Use arithmetic operations with fractions, decimals, and integers appropriately, as required in a given context;
9. Recognize connections among the operations of arithmetic;
   for example, that certain operations are inverses of each other;
10. Apply rules of arithmetic operations (commutative, associative and distributive properties) correctly;
11. Use the correct order of operations in situations where more than one operation is performed;
12. Interpret exponential notation and use laws of exponents for variables with integer exponents;
13. Estimate the results of numerical computations and judge the reasonableness of the results;
14. Verify the reasonableness of numerical computations and their results in a given context using an appropriate number of significant digits;
15. Recognize and generalize numerical, geometrical, and other patterns;
16. Plot linear and non-linear data, using appropriate scales;
17. Given a verbal, graphical, or algebraic representation of a relationship, express it in a different form as required;
18. Determine whether a relationship is a function;
19. Distinguish among linear, exponential, and power functions;

20. Compare linear and non-linear functions with respect to their rates of change;
21. Interpret the meaning of intercept and slope of a linear function in a given context;
22. Interpret the meaning of the intercepts and vertex of the graph of a quadratic function in a given context;
23. Translate a verbal statement into algebraic language;
24. Translate an algebraic statement into words;
25. Evaluate algebraic expressions for specified values of the variables, including cases where a variable may take on negative or fractional values, and recognize that -x does not have to be negative and 1/x might be greater than 1.
26. Generate and/or recognize equivalent forms of expressions, equations, inequalities, and relations;
27. Determine any non-permissible values for the variable in an algebraic expression;
28. Solve linear equations and inequalities, and systems of linear equations;
29. Solve quadratic and rational equations;
30. Solve linear equations and inequalities involving absolute value;
31. Solve a literal equation for a specified variable, including cases when one or more variable(s) is (are) negative;
32. Model and solve contextualized problems using various representations, such as graphs, tables, and equations;
    determine whether or not the results obtained fit the original context;
33. Recognize and apply properties of parallel and perpendicular lines;
34. Determine whether a given two-dimensional figure is a polygon;
    classify polygons according to the number of sides and how their sides and angles are related;
35. Recognize and apply properties of isosceles and equilateral triangles;
36. Recognize and apply the property that the sum of the angle measures in a triangle is 180 degrees;
37. Classify three-dimensional objects with respect to the nature and number of their faces and angles;
38. Apply properties of three-dimensional objects such as prisms, pyramids, spheres, cylinders or cones in problem solving;
39. Recognize that if geometric objects are similar, then angle measures are preserved and side lengths are proportional;
40. Given two similar geometric figures in a specified ratio, determine the ratios of their perimeters, areas, or volumes;
41. Solve problems involving ratio and proportion in similar triangles;
42. Use congruence and similarity to solve problems involving classes of two- and three-dimensional geometric objects;
43. Use the Pythagorean Theorem to solve problems involving right triangles in various contexts;
44. Describe sizes, positions, and orientations of shapes under transformations such as flips, turns, slides, and scaling;
45. Use units of appropriate size and type to measure angles, perimeter, area, surface area, and volume in problem solving;
46. Determine the circumference/perimeters and the area of triangles, parallelograms, trapezoids, and circles;
47. Determine the surface area and volume of selected prisms, pyramids, spheres and cylinders;
48. Decompose complex shapes into simpler ones to find areas or volumes;
49. Solve problems involving perimeter and area of two-dimensional geometric figures;
50. Solve problems involving surface area, and volume of three-dimensional geometric figures;

51. Solve simple problems involving rates and derived measurements for such attributes as velocity and density;
52. Use graphical representations of data to solve problems;
53. Find, use, and interpret mean, weighted mean, or median as appropriate in the context of a given problem;
54. Use principles of probability to make and test conjectures about the results of experiments and simulations;
55. Compute probabilities for compound events;
56. Construct sample spaces and distributions in simple cases;
57. Use the concepts of conditional probability and independent events in problem solving;
58. Differentiate between inductive and deductive reasoning;
59. Interpret and correctly use connecting words, such as “and”, “or”, and “not”;
60. Use examples and counterexamples to analyze conjectures;
61. Distinguish between “if-then” and “if and only if” statements;
62. Determine whether two statements are logically equivalent;
63. State and interpret correctly the negation of a given statement;
64. Analyze the validity of an argument;
65. Given a relationship defined by a table, graph, or formula, tell whether or not it is a function;
66. Given a representation of a function in any of the following forms: a table of values, a graph, an equation or formula, or a verbal description for a function;
    generate a different form of representation, as required, using function notation if appropriate;
67. For any of the representations named in #2, distinguish between input values and output values, and/or provide an input value associated with a specified output value.
    Evaluate a function given in function notation for a specific value of the variable;
68. Use the graph of a function to tell whether or not it is increasing (decreasing) on a specified interval;
69. For linear functions: transform from the form $ax + by = c$ to slope-intercept form and vice versa;
    given two points, or the slope and y-intercept, find the equation;
70. Given two lines that are graphed on the same set of axes, tell which line has the greater (lesser) slope.
    Interpret the slope of a line as a rate of change. Find the point of intersection for two given lines by graphing or by solving the appropriate system of equations;
71. Determine from their equations whether two lines are parallel, perpendicular, or neither. Write an equation of a line parallel or perpendicular to a given line and which passes through a given point.
    Write the equation of a vertical or horizontal line, given one of its points;
72. Given a word problem or a description of a real-world event, model the situation by constructing an appropriate function or equation.
    Use this to find the information required, and interpret your solution in terms of the original situation.
    Recognize when real-world circumstances are best modeled using piecewise-defined functions;
73. Determine the domain and range of a function by examining the graph, the formula, or the constraints of the situation being modeled;
74. Given the graph of a function $y = g(x)$, sketch any of the following graphs by applying suitable horizontal or vertical shifts, or by a suitable reflection:
    $y = g(x) + k$;
    $y = g(x + k)$;
    $y = -g(x)$;
    $y = g(-x)$;
75. Given the graph of a function $y = f(x)$, sketch the graph of $y = k \bullet f(x)$ or $y = f$

(kx). Identify a graph as a horizontal or vertical stretching or compression of another graph;

76. Identify a graph which can be seen as a sequence of translations, or as a sequence of a translation and a stretching or compression. Given a formula for f(x), write a formula for the function after such a sequence of transformations;
77. Given formulas for the functions f(x) and g(x), find a formula for f(g(x)) or for g(f(x)). Given formulas or tables of values for f(x) and g(x), evaluate f(g(x)) or g(f(x)) for a specified value of x. Given the domains of f(x) and g(x), determine the domains of f(g(x)) and g(f(x));
78. Express a complicated function as a composite of simpler functions;
79. Given graphs of two functions f and g, sketch a graph of f + g or f – g. Given formulas for f and g, find the formulas for f + g and f – g;
80. Use the definition of |x| for any real number x to rewrite a function involving absolute values without using absolute value bars;
81. Determine if a given function is one-to-one, and if it is,
    (a) given a formula for f(x), find a formula for $f^{-1}(x)$;
    (b) given a graph of f(x), sketch the graph of the inverse function;
    (c) evaluate $f^{-1}(b)$ for selected values of b from a given graph or table of values of f(x), or by using the formula;
82. Find the zeros of a quadratic function by factoring, completing the square or the quadratic formula;
83. Transform a quadratic function from standard form to vertex form by completing the square. Use the vertex form to find the maximum or minimum value of a quadratic function, or to sketch the graph without the use of calculator or computer;
84. Given any of the following information, find a formula for a parabola:
    a) the x-intercepts and one other point;
    b) the vertex and one other point;
    c) the y-intercept and two other points;
85. Match power functions of the form $f(x)=x^n$ for n = 1,2,3,4, or 5 with their graphs, without the use of a graphing calculator or computer. Describe the behaviour of the graph of any of these functions when the independent variable is close to zero or very large positively or negatively. Use algebra to find a formula for a power function if you know two data points;
86. Given a formula, determine whether or not it defines a polynomial function, and if it is a polynomial, state the degree;
87. Use the Rational Root Theorem to find the zeros of a given polynomial function;
88. Given a polynomial function, identify the x- and y-intercepts. Determine the behaviour of the graph for large positive and negative values of the independent variable. Use this information to draw a rough sketch of the graph;
89. Given a graph of a polynomial function, find an algebraic expression for the function that might produce the graph, and justify your choices;
90. Given a rational function, identify any zeros or vertical asymptotes, and use the coefficients of the leading terms in the numerator and denominator to predict the behaviour of the graph for large positive and negative values of the independent variable;
91. Given the graph of a rational function, find a plausible algebraic expression for a function that could produce this graph, and justify your choice;
92. Write an exponential function to model a quantity that is growing (or decaying) by a fixed percentage in a given time period. Determine the percentage growth rate from the formula for an exponential function;

93. Given an expression for an exponential function, identify the domain and range, the y-intercept, and the horizontal asymptote, and sketch the graph;
94. Given expressions for two or more exponential functions, determine which function will have the steeper graph;
95. Recognize that exponential and logarithmic functions are inverses of each other. Given one such function, write the appropriate expression for the inverse. Given a logarithmic function, sketch its graph;
96. Relate the properties of logarithms to the corresponding properties of exponents. Apply properties of logarithms to solve logarithmic or exponential equations;
97. Identify an angle given in radians as a real number related to the directed rotation of a ray about its endpoint, such that if a ray with its endpoint at the center of a circle of radius r has rotated through an angle of θ radians in a counter-clockwise direction and the point of intersection of the ray with the circle has traversed an arc length of s, then $\theta = s/r$;
98. Convert between radian measure and degree measure of an angle;
99. Associate a point on the unit circle with a given angle θ, and define the sine and cosine of θ in terms of the coordinates of that point. Conversely, use trigonometric functions to find the coordinates of a point P on the unit circle associated with a given angle θ, or on a circle of any radius;
100. Sketch the graphs of $y = \sin x$ and $y = \cos x$. Label intercepts and x-coordinates of turning points. State the domain, range, and period of the sine and cosine functions;
101. Determine the amplitude, period, midline, and horizontal shift of any function of the form $y = A \sin (t - h) + k$ or $y = A \cos (t - h) + k$, and sketch the graph without using a calculator or computer;
102. Given a sinusoidal graph, fit a suitable function to it. Identify sinusoidal behaviour in real-world situations;
103. Define tan θ, cot θ, sec θ and csc θ in terms of sin θ and cos θ. Sketch the graphs of these functions, and determine the domain, range, and asymptotes for each one. Use the definitions to evaluate these functions for specific values of θ or to solve problems as needed;
104. Recognize and use the Pythagorean relationships among the trigonometric functions to establish identities or to solve applied problems;
105. Solve simple trigonometric equations and use reference angles to get all the solutions;
106. Given a table of values, determine the type of function which best fits the given data: linear, quadratic, exponential, sinusoidal, etc.;
107. Compare power, exponential, and logarithmic functions with respect to the values assumed by the function for large positive or negative values of the independent variable;
108. Recognize an arithmetic sequence, and identify the common difference. Calculate an arbitrary term in a given arithmetic sequence;
109. Recognize a geometric sequence, and identify the common ratio. Calculate an arbitrary term in a given geometric sequence;
110. Recognize an arithmetic or geometric series and calculate S_n if required. Use the sigma notation to write S_n. Recognize an infinite geometric series, and find its sum.